ABSTRACT: For more realistically representing the behavior in smear zone around the vertical drain, a new solution is derived by considering the permeability in smear zone linearly or bi-linearly varied with radial distance. Comparing with the solution of using a single value of permeability in smear zone, it shows that using the average values of permeability in smear zone can not result in the same smear effect as varied one. A single value of permeability which can yield the same smear effect as varied case is theoretical expressed. Further it is indicated that even for the same smear effect, the excess pore pressure distribution in smear zone is different for varied and single value cases. The varied case gives a higher excess pore pressure in smear zone, and therefore, the relatively uniform settlement within unit cell.

RESUME: Pour une représentation plus réaliste du phénomène qui se passe autour d' un drainage vertical, il est proposé une nouvelle solution dérivée du processus de perméabilité linéaire ou bi-linéaire dans une "smear" zone. Cette solution varie selon la longueur du rayon. En comparant à la solution qui consiste à utiliser une valeur unique de perméabilité en "smear" zone, il est démontré que l' usage de différentes valeurs moyennes de perméabilité en "smear" zone ne peut aboutir au même résultat. Une valeur unique de perméabilité qui aboutirait au même effet "smear" que pour le système à valeurs variables est théorique. En outre, il est même indiqué que pour le même effet "smear", la distribution de l' excès de pression des pores, dans une zone "smear" est différente entre les deux systèmes. Le système à valeurs variables donne une plus grande valeur de l' excès de pression des pores, aboutissant ainsi donc à un effet relativement uniforme dans l' unité cellulaire.

1 INTRODUCTION

Several solutions for consolidation of a unit cell (cylinder of soil around a single drain) were developed (Barron 1948; Yoshikuni 1979; Hansbo 1981). In all of these solutions, it was assumed that the unit cell is either uniform (idea case) or divided into two zones, namely, the disturbed zone (smear zone) and undisturbed zone. A single value of permeability was used in each zone. However, experimental results indicate that the degree of disturbance, and therefore the permeability, in smear zone is varied with radial distance from the drain. Based on laboratory experimental results, Onoue (1991) suggested a three zone model, completely disturbed zone, partially disturbed zone, and undisturbed zone. Madhav et al. (1993) reported some test data showing that the permeability in smear zone varies significantly with radial distance. Therefore, it is clear that considering the smear zone as uniform is a very rough approximation.

In order to represent the actual case better, a new theoretical solution for unit cell consolidation is derived assuming the permeability in smear zone is linearly or bi-linearly varied with radial distance from the drain. The effects of varying the permeability in smear zone on excess pore pressure distribution and average degree of consolidation are investigated by comparing with a single value of permeability solution. The practical implication of the new solution is also discussed.

2 SOLUTION OF VARYING PERMEABILITY IN SMEAR ZONE

Considering a case that the permeability in smear zone, \( k_s \), is linearly varied with radial distance, \( r \), as shown in Figure 1, and expressed as:

\[
k_s = k_{so} + c \cdot r
\]  

where: \( k_{so} \) is an imaginal permeability at center of the drain, which can be minus, and \( c \) is a positive constant. Then the radial flow velocity in smear zone, \( v_r \), can be expressed as:

\[
v_r = -\frac{k_{so} + c \cdot r}{\gamma_w} \frac{\partial u}{\partial r}
\]  

where: \( \gamma_w \) is the unit weight of water, and \( u \) is excess pore pressure. Using equal vertical strain assumption as adopted by Hansbo (1981), and considering the mass conservation, we get:

\[
\frac{\partial u}{\partial t} = -\frac{1}{2} \frac{\gamma_w}{k_{so} + c \cdot r} \left( R^2 - r^2 \right) \frac{\partial \varepsilon}{\partial t}
\]  

where \( \varepsilon \) is vertical strain, \( R \) is the radius of unit cell, and \( t \) is time.
Integrating for u by using the boundary condition of when \( r=r_w \) (\( r_w \) is the radius of the drain) \( u(0) = 0 \), and if define the dimensionless parameters \( k_0(\theta) = k_m \), and \( R/r_w = n \), for \( r_w < r < n \) (\( n \) is the radius of smear zone), the u can be expressed as:

\[
u = \frac{Y m R^2}{2 k_0} \left( \frac{1}{r_w} + (m^2 - 1) \ln \frac{m + 1/n}{m + 1/n} - \frac{1}{n} \right) \frac{\partial e}{\partial t}
\]  

(4)

Note: Equation (4) is for \( k_0 = 0 \), and this assumption is used throughout this paper. As a supplementary information, for \( k_0 = 0 \), the expression for u is:

\[
u = \frac{Y m^2 R^2}{2 k_0} \left[ \left( 1 - \frac{r_w}{r} \right) - \frac{r}{n} \right] \frac{\partial e}{\partial t}
\]  

(5)

The excess pore pressure in undisturbed zone, \( u < r < R \), is the same as Hansbo's solution (Hansbo 1981) except the boundary condition at \( r = n \) is different, and can be expressed as follows:

\[
u = \frac{Y m^2 R^2}{2 k_0} \left( \frac{1}{r_w} - \frac{r}{R} - \frac{r^2}{R^2} - \frac{s^2}{n^2} \right) + \frac{k_h}{k_0} \left[ \ln \left( m + s/n \right) - \ln \left( m + 1/n \right) \right] \frac{\partial e}{\partial t}
\]  

(6)

where \( k_h \) is horizontal permeability in undisturbed zone, and \( s = r - r_w \). The average excess pore pressure, \( \bar{u} \), will be:

\[
\bar{u} = \frac{1}{R^2 - r_w^2} \int_0^R u \, dr
\]  

(7)

Introducing the relationship between vertical strain and average excess pore pressure (related to effective stress):

\[
\frac{\partial e}{\partial t} = \frac{1}{D} \frac{\partial \bar{u}}{\partial t}
\]  

(8)

where D is constrained modulus. Then combine Equations (7) and (8), and integrate with boundary condition of \( \bar{u}_{r = 0} = \bar{u}_0 \), we get:

\[
\bar{u} = \bar{u}_0 e^{-\frac{r^2}{D} t}
\]  

(9)

where, \( T_h = (k_0D)/(4R^2 \gamma_w) \). The \( \mu \) can be divided into geometry part and smear effect part. The geometry part as well as well resistance part (not derived here) are the same as Hansbo's solution (Hansbo 1981). The expression for smear effect is as follows:

\[
\mu = \frac{k_h}{k_0} \left[ \ln \left( 1 - \frac{m^2}{n^2} \right) \ln \frac{m + s/n}{m + 1/n} + \frac{s - 1}{2 n^2} \left( m^2 - 1 \right) - \left( 2 m^2 - 1 \right) \right]
\]  

(10)

For \( m > 1 \), the permeability variation in smear zone is very small, and the solution will be close to Hansbo's solution. For \( m < 1 \), the smear effect part can be simplified by ignoring the minor terms:

\[
\mu = \frac{k_h}{k_0} \left[ \ln \left( 1 - \frac{m^2}{n^2} \right) \ln \frac{m + s/n}{m + 1/n} + \frac{s - 1}{n} \left( m^2 - 2 \right) - \ln \right]
\]  

(11)

Considering the permeability in smear zone linearly varying is a better approximation than using a single value case. However, further improvement can be obtained by bi-linear approach, i.e., 3 zone model. Figure 2 illustrates the permeability variation for 3 zone model. For this case, the excess pore pressure can be expressed as:

\[ u = \frac{Y m^2 R^2}{2 k_0} \left[ \ln \left( \frac{r}{r_w} + (m^2 - 1) \ln \frac{m + 1/n}{m + 1/n} - \frac{1}{n} \right) \right] \frac{\partial e}{\partial t}
\]  

(12)

(1) for smear zone 1, \( r_w < r < n \):

\[ u = \frac{Y m^2 R^2}{2 k_0} \left[ \ln \left( \frac{r}{r_w} + (m^2 - 1) \ln \frac{m + s/n}{m + 1/n} - \frac{1}{n} \right) \right] \frac{\partial e}{\partial t}
\]  

(13)

(2) for smear zone 2, \( n < r < R \):

\[ u = \frac{Y m^2 R^2}{2 k_0} \left[ \ln \left( \frac{r}{n} + (m^2 - 1) \ln \frac{m + s/n}{m + 1/n} - \frac{1}{n} \right) \right] \frac{\partial e}{\partial t}
\]  

(14)

(3) for undisturbed zone, \( n < r < R \):

\[ u = \frac{Y m^2 R^2}{2 k_0} \left[ \ln \left( \frac{r}{n} + (m^2 - 1) \ln \frac{m + s/n}{m + 1/n} - \frac{1}{n} \right) \right] \frac{\partial e}{\partial t}
\]  

The meaning of the parameters are indicated in Figure 2 also, where \( m_1, m_2, s_1, s_2 \) are dimensionless parameters. Using the same procedure, the expression for smear effect can be obtained. However, due to the limitation of space, it is omitted here.

![Figure 2. Bi-linearly variation model](image)

3 COMPARE WITH HANOSBO'S SOLUTION

In order to compare with Hansbo's solution (Hansbo 1981), a single value of permeability in smear zone must be defined. Three definitions are considered: (1) average value weighted by area, defined as area average \( (k_{a1}) \), (2) the average value of \( k_{aw} \) (permeability for \( r = 0 \)), and \( k_h, k_h, k_{aw} = (k_{aw} + k_{aw})/2 \), defined as simple average, and (3) a value results in the same smear effect as varied one. For linearly varied case, the area average can be expressed as:

\[ k_{aw} = k_0 + \frac{2k_m s^2 + 1}{3} \left( \frac{k_h}{k_m} - 1 \right)
\]  

(15)

The average permeability \( (k_{aw}) \) for resulting in the same smear effect as linearly varied case is a function of \( s, n, m \), and \( k_0 \):
Refer to the field installation condition, assuming \( R = 1.0 \) m, \( t_w = 0.025 \) m, \( t_s = 0.15 \) m, and for \( k_s/k_{rw} = 5 \) and 10, the linearly varied case is compared with the single value of permeability case. For a clear picture, the corresponding permeability ratio is summarized in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Summary of permeability ratio.</th>
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<tbody>
<tr>
<td>( k_s/k_{rw} )</td>
</tr>
<tr>
<td>-----------------------------------------</td>
</tr>
<tr>
<td>5</td>
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<td>10</td>
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The comparison of excess pore pressure distribution is shown in Figures 3 (a) and (b). In the Figure, the excess pore pressure is normalized by the value at periphery \( (\psi_{ur}) \). It can be seen that: (1) varying permeability case results in a smooth excess pore pressure variation within unit cell; (2) even for an average value which yields the same smear effect as varied case, the excess pore pressure in smear zone is lower than linearly varied case; and (3) the area and simple average values under-evaluate the smear effect, and the larger the \( k_s/k_{rw} \), the more under-evaluate will be (compare Figures 3 (a) and (b)). For the case investigated, the single value of permeability for resulting in the same smear effect is corresponding to a value at \( n = 2.0 \) on linear variation line, the larger the \( k_s/k_{rw} \), the smaller this value will be. This implies that in order to determine a single value of permeability in smear zone for resulting in the same smear effect as varied case, a soil sample very close to the drain should be used, otherwise, the smear effect will be under-evaluated.

For illustrating the difference between linear and bi-linear cases, the excess pore pressure distribution is compared under the condition of the same smear effect as shown in Figure 4. The assumed conditions for bi-linear case are: \( k_b/k_{rw} = 10 \), \( k_s/k_{rw} = 5 \), \( k_b/k_{rw} = 2 \), \( k_s/k_{rw} = 1 \) and the corresponding condition for linear case is \( k_b/k_{rw} = 5 \). It can be seen that bi-linear case only shows a slightly higher excess pore pressure in smear zone. Therefore, it is considered that under the same smear effect condition, the linearly variation case can give a result with enough accuracy.

The different excess pore pressure distribution in smear zone will influence the settlement of unit cell. If assuming that constrained modulus, \( D = 1000 \) kPa, horizontal permeability, \( k_h = 10^{-8} \) m/sec, and \( k_b/k_{rw} = 5 \) for linearly varied case, the finite element results indicate that under the same smear effect condition, linearly varying permeability case yields a relatively more uniform settlement distribution than the single value case. If define the settlement ratio as \( \psi_{r} / \psi_{s} \), where \( \psi_{r} \) is the settlement at radius, \( r \), and \( \psi_{s} \) is the settlement at periphery of unit cell. For assumed case, the settlement ratio at the boundary of the drain, \( \psi_{rw}/\psi_{s} \) at \( r = r_w \) is listed in Table 2. It also indicates that under the conditions of considering the smear effect and \( T_h > 0.5 \), the assumption of equal vertical strain seems not introducing much error.

The average degree of consolidation of unit cell is compared in Figures 5 (a) and 5 (b) for \( k_b/k_{rw} = 5 \) and \( k_b/k_{rw} = 10 \) cases, respectively. The same as excess pore pressure distribution, the average values of permeability under-evaluate the smear effect, and the area average value gives highest consolidation rate. This tendency is increased with the increase of \( k_b/k_{rw} \) value (compare

<table>
<thead>
<tr>
<th>Table 2. Settlement ratio.</th>
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<tbody>
<tr>
<td>( T_h )</td>
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<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>0.1</td>
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<tr>
<td>0.5</td>
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<td>1.0</td>
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Figures 5(a) and 5(b)). Noting that although the distance between the curves is small, for a given $T_h$, the maximum difference on the degree of consolidation is about 20%.

4 CONCLUSION

For better representing the actual case, a new unit cell solution is derived by considering the permeability in smear zone linearly or bi-linearly varied with radial distance. The solution can yield smooth excess pore pressure distribution in smear zone, and a relatively more uniform settlement within a unit cell.

Comparing with the solution of using a single value of permeability in smear zone, it indicates that using the average values of permeability in smear zone will under-evaluate the smear effect. A single value of permeability for resulting in the same smear effect as linearly varied one is theoretically expressed. Practically, the permeability in smear zone should be determined by using the soil sample very close to the drain.

REFERENCES