DESIGN METHODS OF PVD INSTALLATION DEPTH FOR TWO-WAY DRAINAGE DEPOSIT

J.-C. Chai¹, N. Miura², T. Kirekawa³ and T. Hino⁴

ABSTRACT: For a two-way drainage deposit under a surcharge load, due to the vertical drainage capacity of a natural deposit, it is possible to leave a layer adjacent to the bottom drainage boundary without prefabricated vertical drain (PVD) improvement and achieve approximately the same degree of consolidation as a fully penetrated case. This depth is designated as an optimum PVD installation depth under a surcharge load. Further, for a two-way drainage deposit under a vacuum pressure, if the PVDs are fully penetrated through the deposit, the vacuum pressure will leak through the bottom drainage boundary. In this case, the PVDs have to be partially penetrated, and there is an optimum installation depth which resulting the maximum consolidation settlement. The equations for calculating these optimum installation depths are presented, and the usefulness of the equations is studied by using one-dimensional finite element analysis as well as laboratory test results.

Keywords: prefabricated vertical drain (PVD), ground improvement, vacuum consolidation, laboratory test, pre-loading, soft clayey deposit

INTRODUCTION

Preloading a soft clayey deposit by a surcharge load and/or vacuum pressure with combination of prefabricated vertical drains (PVDs) is a commonly used and economic ground improvement technique. For a one-way drainage deposit, to shorten the preloading period, normally PVDs are fully penetrated into the deposit. However, for a two-way drainage deposit (a sand layer underlying a soft clayey layer), fully penetration PVDs into the deposit may not be an economic choice. Theoretically, for one-dimensional (1D) consolidation, the rate of consolidation is proportional to \(1/H^2\) (\(H\) is the drainage length), and for a clayey layer with a small drainage length, without PVDs the rate of consolidation can be as fast as the layer with PVDs. Therefore, for a preloading with surcharge load, it is possible to design a PVD improvement with partial penetration to achieve approximately the same rate of consolidation as that of a fully penetrated case. This penetration depth will be called optimum installation depth for surcharge load (\(H_1\)), and the corresponding optimum thickness of PVD unimproved layer is designated as \(H_c\). Although there are several consolidate on solutions have been proposed for PVD improved subsoil (e.g. Hansbo, 1981), there is no rational method proposed for determining \(H_c\).

In case of vacuum consolidation, if PVDs are fully penetrated into a two-way drainage deposit, the vacuum pressure will leak through the bottom drainage boundary and cannot achieve a desired consolidation effect. In this case, PVDs have to be partially penetrated into the deposit and there is an optimum installation depth for vacuum pressure (\(H_v\)) at which the maximum consolidation settlement can be obtained (Chai et al. 2006).

In this paper, an semi-theoretical equation for determining \(H_s\) for surcharge load is derived and the method for determining \(H_v\) is briefly reviewed. Then the effectiveness of the methods is studied by 1D finite element analysis (FEA) as well as some laboratory tests.

PVD INSTALLATION DEPTH FOR SURCHARGE LOAD

Equation for calculating the optimum thickness of PVD unimproved layer (\(H_c\))

The condition for deriving an equation for calculating \(H_c\) (Fig. 1) is that the average degree of consolidation of

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layer $H_c$ under one-way drainage is the same as the average degree of consolidation of the deposit with fully penetration of PVDs. Considering both the drainage capacity of natural deposit and the effect of PVDs, the average degree of consolidation of a PVD improved deposit can be calculated as follows (Carrillo 1942):

$$U_{av} = 1 - \left(1 - U_{vl}\right)\left(1 - U_h\right)$$  \hspace{1cm} (1)

where $U_{vl}$ = the average degree of consolidation in the vertical direction, and $U_h$ = the average degree of consolidation due to PVD. To obtain an explicit expression for $H_c$, $U_{vl}$ is calculated by an approximate form of Terzaghi’s 1D consolidation theory (Chai et al. 2001) assuming the deposit is uniform.

$$U_{vl} = 1 - \exp(-C_d \cdot T_{vl})$$  \hspace{1cm} (2)

where $C_d$ = a constant (=3.2), and $T_{vl}$ is the time factor for vertical drainage and can be expressed as follows:

$$T_{vl} = \frac{c_v \cdot t}{(H/2)^2}$$  \hspace{1cm} (3)

where $c_v$ = the coefficient of consolidation in the vertical direction of the deposit, $t$ = time and $H$ = the thickness of the clayey deposit with two-way drainage. Then, the average degree of consolidation due to PVD ($U_h$) can be evaluated by Hansbo’s solution (1981) as follows:

$$U_h = 1 - \exp\left(-\frac{8T_h}{\mu}\right)$$  \hspace{1cm} (4)

where $T_h$ = time factor for PVD consolidation and $\mu$ = a parameter representing the effect of PVD spacing, smear and well resistance and it can be expressed as follows:

$$T_h = \frac{c_h \cdot t}{D_e \pi^2}$$  \hspace{1cm} (5)

$$\mu = \ln\frac{D_e}{d_w} + \frac{k_h}{k_v} \cdot \ln\frac{d_w}{d_v} - \frac{3}{4} + \frac{2(H/2)^2 \cdot k_h}{3q_w}$$  \hspace{1cm} (6)

where $c_h$ = the coefficient of consolidation in the horizontal direction of the deposit, $D_e$ = the diameter of a unit cell (a PVD with its improvement area), $d_v$ = the diameter of smear zone, $d_w$ = the equivalent diameter of PVD, $k_h$, $k_v$ = the horizontal hydraulic conductivities of natural soil and the smear zone, respectively, and $q_w$ = the discharge capacity of PVD.

For unimproved layer ($H_c$), the average degree of consolidation ($U_{vl}$) can be calculated by Eq. (2) with corresponding time factor, $T_{vl} = \frac{c_v \cdot t}{H_c^2}$. By equating the average degree of consolidation of PVD unimproved layer ($H_c$) and the average degree of consolidation if the deposit is fully penetrated by PVDs, $H_c$ can be expressed as follows:

$$H_c = \frac{1}{\sqrt{\frac{4}{H^2} + \frac{2.5}{\mu \cdot D_e^2} \cdot \frac{c_h}{c_v}}}$$  \hspace{1cm} (7)

$H_c$ calculated from Eq. (7) ensures that the average degree of consolidation of the unimproved layer ($H_c$) under one-way drainage is the same as the average degree of consolidation of the deposit with fully penetration of PVDs. However, with the arrangement as shown in Fig. 1, for the PVD improved layer ($H - H_c$), its drainage length is longer than that of PVD fully penetrated case. Theoretically, this increase of drainage length will increase the well resistance for PVD consolidation and reduce the rate of consolidation due to the vertical drainage of a natural deposit. For most practical cases, when the discharge capacity ($q_w$) of PVD is larger than 100 m$^3$/year, the effect of well resistance is not significant (Bergado et al. 1996). Then this increase of drainage length will mainly reduce the consolidation effect of natural deposit in the vertical direction, and this effect will increase with the increase of PVD spacing, smear effect, and increase of the thickness of a deposit,
i.e. reduce the relative importance of PVD consolidation to the overall average degree of consolidation of the PVD improved layer. To compensate the effect of the longer drainage length of PVD improved layer, the average degree of consolidation of unimproved layer \(H_c\) must be higher than that of PVD fully penetrated case. With above reasoning and by few tries, it has been found that if calculating \(H_c\) by the condition that the average degree of consolidation of unimproved layer is the same as that of PVD fully penetrated plus assuming the drainage length of a natural deposit in the vertical direction is \(H/8\) \((H = \text{the thickness of a two-way drainage deposit})\), the partially penetrated case will result in almost the same average degree of consolidation of a fully penetrated case. This condition results in a modified form of Eq. (7), which is the final equation for calculating \(H_c\).

\[
H_c = \frac{1}{\sqrt{\frac{64}{H^2} + \frac{2.5}{\mu \cdot D_v^2} \cdot \frac{c_h}{c_v}}}
\]  
(7a)

Although Eqs (7) and (7a) are derived assuming the deposit is uniform, the most natural deposit is non-uniform. It is proposed that Eqs (7) and (7a) can be used for non-uniform deposit. The validity of this proposal will be checked in the following sections.

1D finite element analysis (FEA)

The usefulness of Eq. (7a) is checked by 1D FEA. The ground condition at Kubota, Saga, Japan is considered. An about 9 m thick soft Ariake clay layer underlain a sand layer forms a two-way drainage deposit. The subsoil profiles and some of soil properties are given in Fig. 2. In the figure, \(p_c\) is consolidation yielding stress, \(\sigma'_v\) is effective overburden pressure, and OCR is over-consolidation ratio.

### Table 1 Selected parameters for consolidation analysis

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>(c_h) (m²/day)</th>
<th>(c_v) (m²/day)</th>
<th>(\gamma) (kN/m³)</th>
<th>OCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0~1.0</td>
<td>0.030</td>
<td>0.030</td>
<td>16.0</td>
<td>1.00 1.54 4.0</td>
</tr>
<tr>
<td>1.0~3.0</td>
<td>0.021</td>
<td>0.014</td>
<td>14.6</td>
<td>1.38 2.28 2.0</td>
</tr>
<tr>
<td>3.0~6.0</td>
<td>0.036</td>
<td>0.024</td>
<td>14.3</td>
<td>1.07 2.50 1.2</td>
</tr>
<tr>
<td>6.0~9.0</td>
<td>0.048</td>
<td>0.032</td>
<td>14.0</td>
<td>1.62 2.75 1.1</td>
</tr>
</tbody>
</table>

Note: \(\gamma\) = total unit weight; \(C_c\) = compression index; \(e_0\) = initial void ratio; OCR = over-consolidation ratio.
Table 2 Parameters related to PVD consolidation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Basic value</th>
<th>Variation range</th>
</tr>
</thead>
<tbody>
<tr>
<td>PVD spacing</td>
<td>S</td>
<td>m</td>
<td>1.2</td>
<td>0.8, 1.0, 1.2, 1.5, 2.0</td>
</tr>
<tr>
<td>Diameter of unit cell</td>
<td>Dc</td>
<td>m</td>
<td>1.36</td>
<td>0.90, 1.13, 1.36, 1.70, 2.26</td>
</tr>
<tr>
<td>Diameter of drain</td>
<td>dw</td>
<td>mm</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>Diameter of smear zone</td>
<td>ds</td>
<td>m</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td>Discharge capacity</td>
<td>q_w</td>
<td>m³/year</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>Hydraulic conductivity ratio</td>
<td>k_s/k_h</td>
<td>-</td>
<td>10</td>
<td>1, 5, 10</td>
</tr>
</tbody>
</table>

Calculation, a value of $c_h/c_v = 1.5$ is assumed, and $k_h$ value is estimated from $c_s$, $C_v$, $e_0$ and initial effective stress in the ground (assuming the groundwater level is 1.0 m below the ground surface) and averaged with a weight of layer thickness. It is clearly indicated that the over-prediction of $H_c$ by Eq. (7) increases with the increase of the smear effect and PVD spacing, i.e. reduction of the relative weight of PVD consolidation to the overall average degree of consolidation of the PVD improved layer as mentioned previously. Then 1D FEA are conducted to investigate the effect of $H_c$ on settlement rate of the deposit and the analysis results are compared with the $H_c$ values given in Fig. 3.

In the analysis, the effect of PVD is modeled by the method of equivalent vertical hydraulic conductivity (Chai et al. 2001). For PVD spacing $S = 1.2$ m (square pattern) and $k_h/k_s = 10$ case, the effect of $H_c$ on settlement ($S$ – time ($t$)) curves are compared in Fig. 4. It can be seen that for $H_c = 1.0$ m, the settlement curves are almost identical to that of the fully penetrated case. $H_c$ value from Eq. (7a) is 1.05 m (Fig. 3) and it agrees with the FEA results.

Under the condition of $k_h/k_s = 10$, Fig. 5 compares settlement curves of PVD fully penetrated cases with that of partially penetrated cases with $H_c$ values from Eq. (7a) for $S = 0.8$ m, 1.5 m and 2.0 m. It can be seen that the settlement curves of partially penetrated cases are very close to those of the fully penetrated cases. It is not clear, but the difference increases with the increase of PVD spacing. Figure 6 shows the comparison of the settlement curves of PVD fully and partially penetrated cases with $S = 1.2$ m and different $k_h/k_s$ values. Again, the partially penetrated cases yield settlement curves very close to that of the corresponding fully penetrated cases, and the difference increases with the increase of smear effect.

The comparisons made in Figs 4 to 6 use the real soft clay deposit at Kubota, Saga, Japan by assuming different PVD penetrated depth, spacing and smear effect. To further verify the effectiveness of Eq. (7a), it is assumed that the thickness of the soft layer doubles from 9 m to 18 m (double the thickness of each sub-layer). In this case, Eq. (7a) results in an $H_c$ value of 1.81 m ($k_h/k_s = 10$, $S = 1.2$ m). Figure 7 compares the
settlement curves from FEA results. Although the curve of the partially penetrated case is very close to that of the fully penetrated case, in this case the difference is larger than that in Figs 5 and 6. This comparison indicates that with a $H_s$ value calculated by Eq. (7a), the rate of consolidation will increase with the increase of the thickness of a soft clayey deposit. However, for the conditions investigated, it can be said that Eq. (7a) is applicable.

The comparisons made above clearly demonstrate that for a two-way drainage clayey deposit, leaving a layer without PVD improvement with a thickness of $H_c$ calculated by Eq. (7a), the rate of consolidation is very close to that of PVD fully penetrated case. This finding has a practical implication that for a two-way drainage clayey deposit, PVD improvement can be designed more economically. For the subsoil condition at Kubota, Saga, Japan, PVD partially penetration case can save about 15 to 20% of PVD improvement cost. Also, leaving a layer without PVDs adjacent to sand layer has a geoenvironmental benefit, i.e. the water squeezed out from the clayey layer collected by PVDs will not directly into the sand layer (normally an aquifer).

**PVD INSTALLATION DEPTH FOR VACUUM PRESSURE**

Equation for calculating optimum installation depth ($H_1$)

Assuming that the static water table is at the ground surface, the hydraulic conductivities are uniform in both PVD-improved zone and unimproved zone respectively, and for this kind of two-layer system, at steady state the vacuum pressure distribution in the ground can be illustrated as in Fig. 8, and due to the total head difference at the bottom and the top of the soft clayey deposit with a vacuum pressure applied at the ground surface, there will be upward steady flow. Based on: (1) the condition of flow continuity in this two-layer system at steady state, and (2) maximizing the area $A$ (the area between the horizontal and the vertical axes and the vacuum pressure distribution line) as indicated in Fig. 8, an equation for calculating the optimum PVD installation depth ($H_1$), which will results in maximum consolidation settlement, has been derived by Chai et al. (2006).

$$H_1 = \left( \frac{k_{v1} - \sqrt{k_{v1}k_{v2}}}{k_{v1} - k_{v2}} \right) H$$ (8)
where $k_{v1}$ and $k_{v2}$ = the vertical hydraulic conductivities of PVD-improved and unimproved zones, respectively. The value of $k_{v1}$ can be evaluated by the equivalent vertical hydraulic conductivity method (Chai et al., 2001) as follows:

$$k_{v1} = \left(1 + \frac{2.5l^2}{\mu D_e k_v} \right) k_v$$

(9)

where $l$ = the drainage length of the PVDs. It is worth to mention that in field, a clayey deposit may not be uniform. In case the compressibility of the soil layers at around the bottom end of PVDs varies significantly, largest $A$ value may not guarantee a largest final settlement, i.e. Eq. (8) may not yield an optimum PVD installation depth.

Laboratory model test on optimum installation depth $H_1$

(1) Test device

A cylindrical model was used and the sketch of the model is shown in Fig. 9. The model mainly consists of a cylinder of 0.45 m in inner diameter and 0.9 m in height made of vinyl chloride with a wall thickness of 15 mm; upper and lower pedestals with a thickness of 40 mm; a piston system; and a burette connected to the drainage layer at the bottom of the model. The upper and the lower pedestals are connected by eight 12 mm in diameter steel rods. The 40 mm thick piston is made of vinyl chloride and a hollow shaft with an outside diameter of 100 mm is fixed to it. To prevent the possible tilting of the piston, a guide is installed on the upper pedestal around the shaft. Sealing between the piston and the cylinder and between the shaft and the upper pedestal is achieved by “O” rings lubricated by silicon grease. Both air pressure and vacuum pressure can be applied as consolidation load. The air pressure is applied through the upper pedestal to the top of the piston and the vacuum pressure is applied through the hollow shaft of the piston to the bottom of the piston (surface of the model ground). To further prevent the possible air pressure and/or vacuum pressure leakage through the piston, a rubber membrane sleeve with a thickness of 1 mm is installed inside the chamber above the piston. Considering the vertical displacement of the piston during consolidation, the rubber membrane sleeve is folded in vertical direction initially. A KPD-200kPa type piezometer (manufactured by Tokyo Sokki Kenkyuyo Co. Ltd, Japan) is instrumented through the wall of the cylinder. The settlement is measured at the top of the shaft of the piston by a dial gauge.

(2) Materials

The soil used was remolded Ariake clay with a liquid limit $W_l = 108.8\%$, and plastic limit $W_p = 59.2\%$. Initial water content of the clay was adjusted to 120% (more than its liquid limit) and cured in a plastic container for more than 1 day before putting into the model. Mini-PVD was formed by folding non-woven geotextile in 3 layers with a cross-sectional dimension of 30 mm wide and 9 mm in thickness. The geotextile used was made of polypropylene and weighted 131 g/m$^2$. The geotextile was used as drainage layers on the bottom and the top of the model ground also.

(3) Test procedure

To set-up the model test, firstly, the cylinder was installed on the lower pedestal and 3 layers of the geotextile were placed at the bottom of the cylinder as drainage layer. A thin layer of silicon grease was painted on the wall of the cylinder to reduce the friction. Then clay was put inside the cylinder layer by layer and when the thickness of the soil reached 0.4 m from the bottom, the piezometer was installed. The thickness of the model ground was 0.78 m. After the completion of the model ground, 4 mini-PVDs were installed to predetermined depth with the plan locations as indicated in Fig. 9. The method of installing mini-PVD is: put a stainless steel rod inside the mini-PVD and push it into the model ground vertically, and then withdraw the rod and leave the mini-PVD inside the model ground. After that another 3 layers of the geotextile were placed on the top of the model ground as a drainage layer. Finally, the piston, the rubber membrane sleeve, and the upper pedestal were installed and the dial gauge for measuring the vertical displacement was set. The pre-determined air-pressure and vacuum pressure were applied and the test was started.
(4) Test program

Totally 4 tests were conducted with the conditions as listed in Table 3. The mini-PVD installation depth varied from 0.48 m to 0.73 m leaving an unimproved layer of 0.30 to 0.05 m in thickness, respectively. Each test was lasted for about 15 days. It is well known that vacuum consolidation not only induces settlement, but also inward lateral displacement of a ground. For the model described above, if only apply vacuum pressure, there is a possibility of forming gaps between the soil and the model wall due to possible inward lateral displacement and causing leakage of vacuum pressure. To avoid this kind of situation, both surcharge load (air pressure) and vacuum pressure were applied. Chai et al. (2005) reported that the condition for no inward lateral displacement to occur can be expressed as follows:

\[
\Delta \sigma_{\text{vac}} < \frac{K_o \cdot \sigma'_{vo}}{1 - K_0}
\]

(10)

where \( K_o \) = at-rest horizontal earth pressure coefficient; \( \sigma'_{vo} \) = in situ vertical effective stress; and \( \Delta \sigma_{\text{vac}} \) = vacuum pressure. Adopting a \( K_0 \) value of 0.5, \( \Delta \sigma_{\text{vac}} < \Delta \sigma'_{vo} \) can be obtained. Considering the surcharge load as \( \sigma'_{vo} \), a condition of surcharge load equals vacuum pressure (40 kPa) was adopted for all tests.

Test results

The settlement curves are compared in Fig. 10. The final part of the curves is enlarged as an insert figure in Fig. 10. The test results indicate that Case-2 and 3 resulted in larger settlement than Case-1 and 4 (Fig. 10). Figure 11 plots the final settlement at corresponding mini-PVD installation depth. It can be observed that from final settlement point of view, an optimum mini-PVD installation depth exists between Case-2 and 3.

From laboratory incremental load consolidation test results, an initial hydraulic conductivity of the model ground, \( k = 2.51 \times 10^{-9} \text{ m/s} \), can be calculated. To apply Eq. (8) to the model test, the parameters related to the effect of mini-PVDs need to be determined. For the arrangement as shown in Fig. 9, \( D_e = 0.225 \text{ m} \) can be evaluated, and the model ground was formed by remolded Ariake clay and \( k_h/k_s = 1 \). Regarding the diameter (\( d_w \)) and the discharge capacity (\( q_w \)) of the mini-PVDs, by fitting the measured settlement curves by FEA, \( d_w = 10 \text{ mm} \) and \( q_w = 1 \text{ m}^3/\text{year} \) were back-calculated (Chai et al. 2007). Then using Eqs (6), (8) and (9), an optimum mini-PVD installation depth of about 0.575 m can be obtained, which is close to the installation depth of Case-2 of 0.58 m. From Fig. 11, it can be seen that increase the mini-PVD installation depth from 0.58 m to 0.68 m did not increase the final settlement. From an economic point of view, the optimum installation depth should be close to 0.58 m. The above comparison and discussion indicate that Eq. (8) is useful for determining the optimum installation depth of PVDs with two-way drainage condition under a vacuum pressure.

### Table 3 List of the cases tested

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial thickness (m)</th>
<th>Mini-PVD installation depth (m)</th>
<th>Air pressure (kPa)</th>
<th>Vacuum Pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.48 (0.30)*</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>0.78</td>
<td>0.58 (0.20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.68</td>
<td>0.68 (0.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.73</td>
<td>0.73 (0.05)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The number in parenthesis is the thickness of mini-PVD unimproved layer.
CONCLUSIONS

Prefabricated vertical drain (PVD) improvement is commonly employed when preloading a soft clayey deposit by a surcharge load and/or vacuum pressure. In case of a two-way drainage deposit, there are optimum PVD installation depths, and PVDs no need to be fully penetrated through the deposit. These findings have a practical implication that a PVD improvement can be designed more efficiently and economically.

(1) Under a surcharge load: A semi-theoretical equation is derived for calculating the optimum PVD installation depth. A partially penetrated case with the optimum PVD installation depth can result in approximately the same average degree of consolidation as a fully penetrated case. The effectiveness of the proposed equation has been verified by one-dimensional finite element analysis.

(2) Under a vacuum pressure: PVDs have to be partially penetrated into the soft clayey deposit to prevent the vacuum pressure losing from the bottom drainage boundary. An optimum installation depth is existed at which the maximum vacuum consolidation settlement can be obtained. The equation for calculating this optimum installation depth is reviewed, and the usefulness of the equation is confirmed by laboratory model test results.

REFERENCES